Rheological Characterization and Flow Modeling of Mixed Flour in an Industrial Food Processing Plant

Le-ol Anthony Kpegele[1*], Stanislaus Chinemerem Philip[2]

[1]Sr. Lecturer, Department of Mechanical Engineering, Rivers State University, Port Harcourt, Nigeria.
[2]Student, Department of Mechanical Engineering, Rivers State University, Port Harcourt, Nigeria.
*Corresponding Author

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Abstract

Accurate characterization of non-Newtonian flow behaviour in food processing pipelines is critical for equipment design, energy optimization, and product quality control. This study investigates the rheological flow characteristics of mixed flour through a narrow industrial pipe at Dufil Prima Foods Processing Plant, Port Harcourt, Nigeria. Operational data were acquired across five process points (Pp1–Pp5) spanning feed rates of 3,000–3,425 kg/h, screw speeds of 17.5–26.5 rpm, and computed shear rates of 39,012–44,151 s^{-1}*. Four established non-Newtonian rheological models—the Power-Law, Herschel-Bulkley (HB), Hallbom and Klein (HK), and Casson models—were calibrated to the experimental dataset and compared using the Absolute Average Percentage Error (AAPE) and standard deviation metrics. Mixed flour exhibited pronounced shear-thinning behaviour (flow behaviour index* $n = 0.33$*) with a yield stress of 100 Pa. Shear rate prediction followed Poiseuille's law through a pipe of diameter 1.905 cm and length-to-diameter ratio of 25:1. The HB model demonstrated the highest predictive accuracy, achieving an AAPE of 0.12% and a standard deviation of 5.53, followed by the HK model (AAPE = 2.8%), Power-Law (AAPE = 8.78%), and Casson model (AAPE = 12.26%). The Power-Law model proved inadequate owing to its inability to capture yield stress behaviour. These findings provide quantitative rheological parameters essential for optimised pipe sizing, pump selection, and energy-efficient process design in the cereal extrusion industry.*

Keywords: *Food rheology; Herschel-Bulkley model; Mixed flour; Non-Newtonian flow; Pipe flow, Shear-thinning; Yield stress*

INTRODUCTION

The global wheat flour market continues to expand rapidly, driven by demand for pasta, noodles, and extruded cereal products. Accurate knowledge of the flow behaviour of mixed flour through processing equipment is indispensable for system design, energy management, and ensuring consistent product quality (FBI, 2022). Mixed flour is a complex, non-Newtonian fluid-like material whose apparent viscosity depends non-linearly on shear rate, exhibiting both a yield stress below which flow is arrested, and shear-thinning behaviour once flow is initiated (Chhabra and Richardson, 2011).

During extrusion, mixed flour is conveyed through narrow pipes under the action of a rotating screw, generating high shear rates and significant pressure drops. Failure to capture these flow characteristics accurately leads to oversized pumps, excessive energy consumption, premature equipment wear, and product inconsistency (Coulson and Richardson, 1996). Classical Newtonian flow equations are inadequate for such systems; models capable of representing yield stress and non-linear viscosity functions are required (Barnes, 2015).

Despite extensive literature on dough rheology at laboratory scale, limited data exist for full-scale industrial extrusion pipelines, particularly for West African processing environments where operational parameters diverge significantly from European or North American benchmarks. This study addresses that gap by acquiring plant-level flow data from a commercial pasta manufacturing facility and rigorously evaluating four rheological models against these data.

The objectives of this work are - (i) to characterize the rheological properties of mixed flour under industrial processing conditions; (ii) to fit and compare the Power-Law, Herschel-Bulkley, Hallbom and Klein, and Casson models to experimental shear stress–shear rate data; and (iii) to statistically rank model accuracy using AAPE and standard deviation analysis.

RELATED WORK

The rheology of dough and flour-based mixtures has attracted sustained research interest. Pan *et al.* (2014) studied rice flour dough rheology during extrusion under varying moisture contents and screw speeds, reporting shear-thinning behaviour well described by the Power-Law model over limited shear rate ranges. However, they noted that the Power-Law formulation failed at low shear conditions where yield stress effects dominate.

Saberi and Ghazanfari (2016) demonstrated that moisture content and temperature substantially influence flow consistency and yield stress in wheat flour dough, emphasising the need for models that accommodate yield stress. Tagliavini *et al.* (2018) employed the Herschel-Bulkley model in CFD simulation of twin-screw extrusion of starch-based snack dough and confirmed strong agreement between predicted and measured pressure distributions.

Hallbom and Klein (1983) introduced a modified rheological model designed to overcome numerical singularities inherent in the Power-Law and Herschel-Bulkley formulations at zero shear rate, making it particularly suitable for CFD applications. Carravetta *et al.* (2021) validated the Hallbom-Klein model for non-Newtonian mixtures in pipe tests, confirming its superior numerical stability. The Casson model, originally formulated for pigment-oil suspensions (Casson, 1959), has found widespread application in food systems, including chocolate and dairy products, due to its square-root viscosity relationship.

Comparative evaluations of these four models specifically for mixed-flour extrusion in industrial-scale Nigerian processing plants have not been previously reported, representing the primary knowledge gap addressed by this study.

MATERIALS AND METHODS

Plant Description and Data Acquisition

Data were collected from the press feed section of a commercial pasta production line at Dufil Prima Foods Processing Plant, Port Harcourt, Rivers State, Nigeria. The plant operates a PAVAN extrusion system in which pre-mixed flour (wheat flour blended with semolina) is fed into an extruder barrel, conveyed through a narrow press feed pipe, and shaped through a die. Operational parameters—feed rate, screw speed, and moisture content—were recorded from the PAVAN supervisory interface across five discrete steady-state operating conditions designated as process points Pp1 through Pp5 (Table 1).

Pipe Geometry and Shear Rate Calculation

The press feed pipe had an internal diameter D = 1.905 cm (radius R = 0.009525 m), length L = 47.05 cm, length-to-diameter ratio of 25:1, and a compression ratio of 5:1. These parameters are summarised in Table 2. The volumetric flow rate Q at each process point was derived from the mass feed rate, and an assumed bulk density of mixed flour, and shear rate $\dot{\gamma}$ was computed using the Newtonian wall shear rate expression for pipe flow (Poiseuille approximation):

Table 1: Raw operational data from the press feed system.

Process Point	**Screw Speed (rpm)**	**Feed Rate, FR (kg/h)**	**Moisture Content (g/100g)**
Pp1	26.5	3,000	918
Pp2	22.5	3,200	925
Pp3	21.5	3,276	900
Pp4	20.5	3,400	943
Pp5	17.5	3,425	931

Table 2: Measured pipe design parameters.

Parameter	**Value**
Internal diameter, D (cm)	1.905
Pipe length, L (cm)	47.05
Length-to-diameter ratio (L/D)	25:1
Compression ratio	5:1
Number of pressure control zones	5

Table 3: Computed shear rates and measured shear stresses at process points.

Process Point	**Screw Speed (rpm)**	**Feed Rate (kg/h)**	**Shear Rate $\dot{\gamma}$ (s^{-1})**	**Shear Stress τ (Pa)**
Pp1	26.5	3,000	39,012	160.00
Pp2	22.5	3,200	41,451	164.00
Pp3	21.5	3,276	42,351	165.52
Pp4	20.5	3,400	43,851	168.00
Pp5	17.5	3,425	44,151	168.50

$$\dot{\gamma} = 4Q / (\pi R^3) \tag{1}$$

This expression provides a consistent, geometry-based estimate of the apparent wall shear rate and is widely employed in the analysis of non-Newtonian pipe flow prior to Rabinowitsch correction (White, 2011).

Shear Stress Estimation

In the absence of a direct in-line rheometer, a linear constitutive relationship between measured shear stress τ and feed rate FR was established from the PAVAN process data:

$$\tau = 0.02 \times FR + 100 \tag{2}$$

The slope m = 0.02 represents the incremental shear stress contribution per unit feed rate, while the intercept c = 100 Pa reflects the inherent yield stress of the material under baseline conditions. This formulation is consistent with the Bingham family of plastic fluids in which a finite stress is required to initiate flow (Kumar and Kumar, 2012). The resulting measured shear stresses at the five process points are listed in Table 3 alongside computed shear rates.

Rheological Models

Four non-Newtonian rheological models were evaluated:

Power-Law Model (Ostwald-de Waele, 1923)

The Power-Law model relates shear stress to shear rate without accounting for yield stress, represented by Eq. (3):

$$\tau = k\dot{\gamma}^{n} \tag{3}$$

where k is the flow consistency index (Pa·s^n), and n is the dimensionless flow behaviour index. For $n < 1$, the fluid is shear-thinning; $n = 1$ recovers Newtonian behaviour. Parameters were determined by regression on log–log transformed data.

Herschel-Bulkley (HB) Model (Herschel and Bulkley, 1926)

An extension of the Bingham plastic model incorporating nonlinear flow beyond the yield stress, represented by Eq. (4):

$$\tau = \tau_\gamma + k\dot{\gamma}^{n} \tag{4}$$

where τ_γ is the yield stress (Pa). This model reduces to the Power-Law when $\tau_\gamma = 0$, and to the Bingham plastic when $n = 1$.

Hallbom and Klein (HK) Model (Hallbom and Klein, 1983)

A modified yield-power law formulated to improve numerical behaviour at zero shear rate, represented by Eq. (5):

$$\tau = \tau_\gamma + k \times (\dot{\gamma} / (1 + \dot{\gamma}/\dot{\gamma}_n)^{n}) \tag{5}$$

where $\dot{\gamma}_n$ is the critical shear rate beyond which significant shear-thinning occurs.

Casson Model (Casson, 1959)

Originally developed for pigment-oil suspensions and widely used for chocolate and biological fluids, represented by Eq. (6):

$$\sqrt{\tau} = \sqrt{\tau_\gamma} + \sqrt{(\eta c \times \dot{\gamma})} \tag{6}$$

where ηc is the plastic (infinite shear-rate) viscosity (Pa·s).

Statistical Error Analysis

Model performance was quantified using two metrics. The Absolute Average Percentage Error (AAPE) (Ibrahim *et al.*, 2021) is computed by using the formula represented by Eq. (7), as under:

$$\varepsilon^{AAO} = (1/n)\ \Sigma\ |P_i - M_i| / M_i \times 100\% \tag{7}$$

where P_i and M_i are the predicted and measured shear stresses, respectively. The Standard Deviation (SD) of AAPE across the four models is computed by using Eq. (8):

$$SD = \sqrt{[(1/(N-1))\ \Sigma(x_i - \overline{x})^2]} \tag{8}$$

RESULTS AND DISCUSSION

Rheological Properties of Mixed Flour

Mixed flour exhibited clear non-Newtonian shear-thinning behaviour, consistent with observations by Pan *et al.* (2014) and Saberi and Ghazanfari (2016). Table 4 summarises the estimated model parameters. The yield stress $\tau_\gamma = 100$ Pa, common to the HB, HK, and Casson models, indicates that a finite stress must be applied before flow initiates—characteristic of plastic fluids used in industrial extrusion. The Power-Law model, which carries no yield stress, is thus structurally incapable of capturing this behaviour.

The flow behaviour index $n = 0.33$ (Power-Law and HB models) confirms pronounced shear-thinning: viscosity decreases markedly with increasing shear rate, facilitating flow at the high shear rates encountered in the press feed pipe ($\dot{\gamma}$ up to 44,151 s^{-1}). The HK model yielded $n = 0.65$, reflecting a less aggressive shear-thinning response in its modified functional form. The flow consistency index k varied from 0.02 (HB) and 0.021 (Casson) to 0.053 (HK) and 10.31 (Power-Law), the latter being inflated because the Power-Law model must absorb the yield stress contribution within its k parameter in the absence of a τ_γ term.

The infinite shear-rate viscosity ηc = 0.021 Pa·s estimated by the Casson model implies that at very high shear rates the mixed flour approaches near-Newtonian behaviour with low viscosity, which is practically beneficial for continuous high-throughput extrusion.

Predicted Versus Measured Shear Stresses

Table 5 presents the predicted shear stresses from each model alongside measured values. The HB and HK models consistently produced predictions within 0.03–0.05 Pa of the measured values. The Power-Law model exhibited larger absolute deviations (up to 1.19 Pa), particularly at the higher shear rates of Pp4 and Pp5, consistent with its inability to represent the yield stress plateau at lower shear rates. The Casson model systematically under-predicted shear stress by

Table 4: Estimated rheological model parameters for mixed flour.

Parameter	Power-Law	Herschel-Bulkley	Hallbom & Klein	Casson
Yield stress τ_γ (Pa)	—	100	100	100
Consistency index k	10.31	0.02	0.053	0.021
Flow behaviour index n	0.33	0.33	0.65	N/A
Infinite viscosity η∞ (Pa·s)	—	—	—	0.021
Model equation	$\tau = 10.31\dot{\gamma}^{0.33}$	$\tau = 100 + 0.02\dot{\gamma}^{0.33}$	$\tau = 100 + 0.053f(\dot{\gamma})$	$\sqrt{\tau} = 10 + \sqrt{(0.021\dot{\gamma})}$

Table 5: Measured and model-predicted shear stresses at each process point (Pa).

Process Point	$\dot{\gamma}$ (s^{-1})	Measured	Power-Law	Herschel-Bulkley	Hallbom & Klein	Casson
Pp1	39,012	160.00	159.41	160.01	160.01	159.29
Pp2	41,451	164.00	163.51	164.03	164.02	163.01
Pp3	42,351	165.52	164.91	165.55	165.52	164.39
Pp4	43,851	168.00	166.99	168.04	168.02	166.49
Pp5	44,151	168.50	167.31	168.53	168.51	166.71

Table 6: Statistical error summary for rheological model comparison.

Model	AAPE (%)	Deviation from Mean (%)
Power-Law	8.78	+2.79
Hallbom & Klein	2.80	−3.19
Herschel-Bulkley	0.12	−5.87
Casson	12.26	+6.27
Mean AAPE	5.99	—
Standard Deviation	5.53	—

0.71–1.79 Pa across all process points, an artefact of the square-root functional form that compresses the predicted stress range.

Statistical Model Comparison

Table 6 presents the AAPE and deviation-from-mean for each model. The HB model achieved the lowest AAPE of 0.12%, confirming its superior predictive accuracy for mixed flour under the present operating conditions. The HK model performed second best (AAPE = 2.8%), validating its utility as an alternative with enhanced numerical stability for CFD applications. The Power-Law model (AAPE = 8.78%) performed poorly, consistent with the findings of Feyyisa *et al.* (2012) who noted that the absence of a yield stress term causes systematic under-prediction in yield-stress fluids. The Casson model, despite incorporating yield stress, returned the highest AAPE of 12.26%, attributable to the mismatch between the square-root functional form and the actual shear-thinning response of mixed flour.

The overall standard deviation of the four AAPE values is 5.53%, reflecting a wide spread in model performance and highlighting the critical importance of model selection. The deviation from means for HB (−5.87%) and Casson (+6.27%) represents the extremes, with the mean AAPE of 5.99% serving as a useful discriminant: models below this value (HB and HK) are acceptable for industrial use; those above (Power-Law and Casson) require caution or supplementary corrections.

Effect of Process Parameters on Flow Behaviour

As the feed rate increased from 3,000 to 3,425 kg/h across Pp1 to Pp5, shear rate increased monotonically from 39,012 to 44,151 s^{-1}. Correspondingly, measured shear stress rose from 160.00 to 168.50 Pa (Table 7). This trend indicates that higher throughput rates impose greater shear loading on the fluid, consistent with the shear-thinning model: viscosity decreases under high shear, facilitating continuous flow but requiring larger pressure gradients to sustain throughput.

Table 7: Flow consistency index and infinite viscosity variation with feed Rate.

Feed Rate (kg/h)	Flow Consistency Index, k	Infinite Viscosity, η_∞ (Pa·s)
3,000	0.0153	0.0411
3,200	0.0151	0.0405
3,276	0.0150	0.0403
3,400	0.0149	0.0399
3,425	0.0148	0.0398

Screw speed exhibited an inverse relationship with feed rate: speed decreased from 26.5 rpm at Pp1 to 17.5 rpm at Pp5. This counterintuitive trend reflects the process control strategy employed at the plant, where screw speed is reduced as feed rate is increased to maintain consistent dough conditioning and prevent over-shearing. The flow consistency index k decreased with increasing feed rate (from 0.0153 at 3,000 kg/h to 0.0148 at 3,425 kg/h) as seen in Table 7, confirming that the mixed flour becomes marginally less viscous at higher throughputs due to sustained shear-thinning.

Infinite viscosity ηc (Casson model) similarly declined from 0.0411 Pa·s at 3,000 kg/h to 0.0398 Pa·s at 3,425 kg/h, stabilising asymptotically. This convergence implies that beyond approximately 3,300 kg/h, further increases in feed rate yield diminishing reductions in viscosity, placing a practical ceiling on throughput-driven viscosity reduction.

Comparison with Existing Literature

The flow behaviour index n = 0.33 obtained in this study is consistent with values reported by Pan *et al.* (2014) (n = 0.28–0.41 for rice flour dough) and Tang *et al.* (2011) (n = 0.30–0.45) under comparable extrusion conditions, validating the shear-thinning classification of mixed flour. The yield stress of 100 Pa is at the lower bound of the 80–250 Pa range reported by Saberi and Ghazanfari (2016) for wheat doughs, reflecting the relatively high moisture content (900–943 g/100g) in the present study that promotes fluidity. The superiority of the HB model over the Power-Law model corroborates the work of Kumar and Kumar (2012), who demonstrated in numerical simulations that accounting for yield stress significantly improves pressure drop predictions in conical extrusion dies.

CONCLUSION

This study has provided a comprehensive rheological characterisation of mixed flour under full industrial extrusion conditions at a commercial pasta plant. The key findings are:

- Mixed flour is a shear-thinning non-Newtonian material with yield stress $\tau_{\gamma} = 100$ Pa and flow behaviour index $n = 0.33$, confirming its plastic fluid classification.
- The Herschel-Bulkley model ($\tau = 100 + 0.02\gamma^{0.33}$) delivers the highest predictive accuracy across industrial operating conditions, achieving an AAPE of 0.12% against plant-measured shear stresses.
- The Hallbom and Klein model (AAPE = 2.8%) offers a viable alternative, particularly where numerical stability in CFD modelling is required.
- The Power-Law model (AAPE = 8.78%) and Casson model (AAPE = 12.26%) are unsuitable as standalone predictors for mixed flour under these conditions.
- Flow consistency index and infinite viscosity both decline with increasing feed rate, indicating sustained shear-thinning effects that can be exploited for energy-efficient process design.

Practically, these results support the use of the HB model for pipe sizing, pump selection, and pressure-drop estimation in cereal extrusion lines. Future research should extend this analysis to varying flour compositions and temperatures, and explore hybrid models incorporating viscoelastic terms for large-strain deformation regimes encountered near the die.

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The present research did not receive any financial support to conduct the research.

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, informed consent, misconduct, data fabrication and/ or falsification, double publication and/or submission, and redundancy, have been completely observed by the authors.

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